

MAT 1348 3X – Practice Test # 1 – Spring/Summer 2016

Nom_____

d'étudiant_____

MAT 1348 3X – Practice Test # 1 – Spring Summer 2016

Name_____

student number_____

| Question | Possible Points | Points Obtained |
|--------------|-----------------|-----------------|
| # 1 | 5 | |
| # 2 | 5 | |
| # 3 | 5 | |
| # 4 | 5 | |
| # 5 | 5 | |
| Total | 30 | |

Instructions:

- Print your name and student number on the first two pages.
- Verify that your copy of the test has all of its 8 pages.
- You must answer all questions. There are 5 questions for a total of 25 points.
- Write the solutions to the questions in the space provided. You may use the back of the pages if necessary.

SHOW ALL YOUR WORK

1. (a) (**3 pts**) Determine the truth table for the following compound propositions:

(i) $((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p))$

| p | q | |
|-----|-----|--|
| | | |

(ii) $((p \rightarrow q) \wedge (\neg p \rightarrow q))$

| p | q | |
|-----|-----|--|
| | | |

- (b) **(1 pt)** Indicate if the above formulas are tautologies, contradictions or contingencies.

(i)

$$(\ddot{\mathbf{i}}\mathbf{i})$$

- (c) **(1 pt)** Are these two formulas logically equivalent? If yes, explain your reasoning, if not, give a counter-example.

2. (5 pts) Are these system specifications consistent?

“The router can send packets to the edge system only if it supports the new address space. For the router to support the new address space it is necessary that the latest software release be installed. The router can send packets to the edge system if the latest software release is installed. The router does not support the new address space.”

3. (**5 pts**) An inhabitant B on the island of knights and knaves is accused of a crime. His lawyer, A, is also an inhabitant of the island. During the proceedings, the following declarations are made:

A : My client is a knave only if he is guilty.

B : My lawyer is a knave if and only if I am innocent.

- (a) Can you determine what the type of inhabitants are A and B? If yes, indicate which is which.
- (b) Can you determine whether B is innocent? If yes, indicate if he is guilty or innocent.

4. (a) (2 pts) Translate the following argument into argument form. Use the following propositional variables:

M : f admits a maximum on $[a, b]$.

C : f is continuous on $(a, b]$.

A : f is defined at $x = a$.

P : There exists a point c between a and b such that $f(c) \geq f(x)$ for all $x \in (a, b]$.

“If f is continuous on $(a, b]$, then f admits a maximum on $[a, b]$ if it is defined at $x = a$. If f is not defined at $x = a$, then there exists a point c between a and b such that $f(c) \geq f(x)$ for all $x \in (a, b]$. There exists a point c between a and b such that $f(c) \geq f(x)$ for all $x \in (a, b]$ only if f admits a maximum on $[a, b]$. Therefore, for f to admit a maximum on $[a, b]$ it is sufficient that f be continuous on $(a, b]$.”

DO NOT VERIFY THE VALIDITY OF THIS ARGUMENT

- (b) (**3 pts**) Verify the validity of the following argument. If the argument is invalid, give all the possible counter-examples.

$$\frac{\begin{array}{l} ((A \wedge B) \rightarrow C) \\ (\neg C \vee D) \\ (\neg D \wedge B) \end{array}}{\neg A}$$

5. (a) (**1 pt**) Identify which of the following expressions are conjunctive clauses.

(i) $(p \wedge \neg p)$

(ii) q

(iii) $(p \wedge \neg q \vee r)$

(b) (**1 pt**) Identify which of the following expressions are in DNF.

(i) p

(ii) $\neg(p \wedge q) \vee (\neg p \wedge q)$

(iii) $(p \wedge \neg q \wedge r) \vee p \vee (q \wedge \neg r)$

(c) (**3 pts**) Consider the truth table for the compound formula ϕ :

| p | q | r | ϕ |
|-----|-----|-----|--------|
| T | T | T | T |
| T | T | F | F |
| T | F | T | F |
| T | F | F | F |
| F | T | T | T |
| F | T | F | F |
| F | F | T | T |
| F | F | F | F |

Find a compound proposition in disjunctive normal form (DNF) equivalent to ϕ .